How Does Root Mean Squared Alternating Voltage Affect Direct Current After Passing Through a Full Bridge Rectifier?

IB Physics HL

Individual Investigation

Page Count: 13

1. Introduction.

One of the main things that interested me in taking physics was circuits and electricity. I grew up playing with a Snap Circuits toy, and it fascinated me to make a fan turn on from a battery. I am also interested in electrical engineering, and have always wanted to create my own electronic devices to solve problems. To learn how to build such devices, I decided to investigate a single circuit: the full bridge rectifier. Specifically, I seek to answer the question: How does root mean squared alternating voltage affect direct current after passing through a full bridge rectifier?

2. Background Information.

In an electrical circuit, the current (in amps) is the rate of electric charge motion that “flows” through conductors, caused by a voltage (difference in potential electrical energy that is measured in volts), and opposed by resistance (in ohms). In alternating current (AC), current and voltage reverse periodically, according to a sine wave. In the United States, power grids supply AC to wall sockets in homes and offices, but almost every electronic appliance requires unidirectional direct current (DC). Rectifiers are specifically made, using diodes, to convert periodical AC power into unidirectional DC.

Diodes only allow unidirectional current flow. Specifically, they are p-n junction diodes, made of a p-type material with excess electrons and an n-type material with excess holes. Excess electrons fill excess holes in a “depletion region” until equilibrium is reached, creating an electric field directed towards the p-side, acting as a potential barrier (AspenCore, n.d.). If a positive external voltage is applied from the p-side anode across to the n-side cathode, the potential barrier is cancelled, and the diode becomes forward biased. However, applied voltage must be greater than the diode’s “forward voltage” for current to flow through. Applying negative external voltage makes the diode reverse biased, reinforcing the potential barrier and preventing most current from flowing across in the reverse direction. Some reverse leakage current still flows through. At a certain reverse breakdown voltage, the diode can fully conduct in the reverse direction (AspenCore, n.d.). Diodes thus mostly conduct unidirectionally.

Whereas a halfwave rectifier converts one half of each AC cycle into positive DC voltage, a full wave bridge rectifier converts all AC half-cycles, and uses four rectifier diodes. Looking at Figure 1, in the positive half of the AC cycle, diodes D1 and D2 are forward biased, while D3 and D4 are reverse biased. They switch in the negative half of the AC cycle. The full bridge rectifier thus only produces unidirectional positive DC voltage, causing DC to flow through the load. This load is shaded in Figure 1:

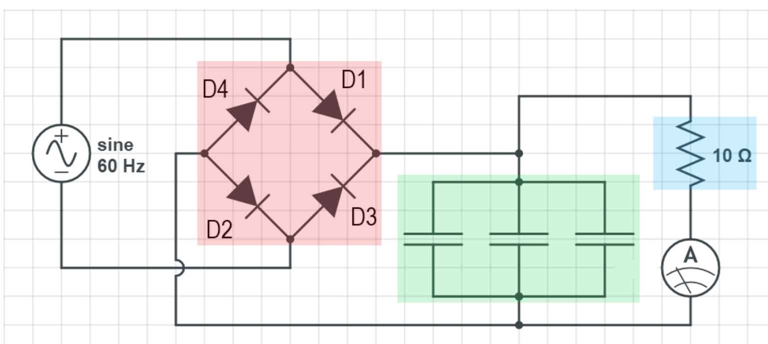


Figure 1: Circuit diagram of the full bridge rectifier used in the experiment. The rectifier diodes are shaded on the left, the smoothing capacitors is shaded in the middle, and the load resistor is shaded on the right. I made this circuit diagram using circuitlab.com.

Although increasing applied voltage past the forward voltage will increase current through a forward biased diode, the current-voltage (I-V) relationship is not linear, but exponential. Thus, diodes are non-Ohmic. This suggests a non-linear I-V relationship in rectifiers. By investigating such a relationship to answer this investigation’s question, rectifiers can be designed to ensure appropriate DC is supplied to electrical devices, depending on the wall socket RMS AC voltage.

3. Variables.

The experiment's dependent variable is the rectifier's DC output, which is measured by the multimeter's DC setting. A rectifier without a capacitor produces positive but oscillating DC output (Figure 2), making measurement difficult. A capacitor connected in parallel to the load can periodically charge and discharge to smooth out the waveform. There is still some ripple voltage, equal to $\frac{I_{load}}{2fC}$, where I_{load} is the DC through the load, f the frequency of the ripple, and C the capacitance (AspenCore, n.d.). To minimize ripple voltage, capacitance C was maximized by connecting three capacitors (with the highest available capacitance of 100 $\mu\mathrm{F}$) in parallel (shaded in Figure 1), making the equivalent capacitance equal to $3(100) = 300\ \mu\mathrm{F}$.

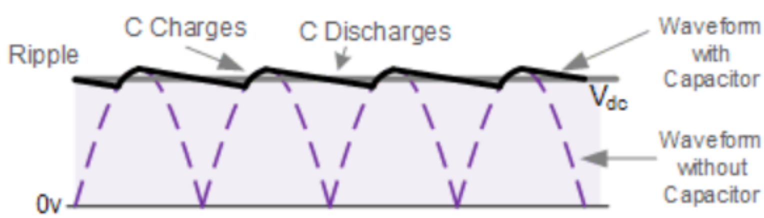


Figure 2: Rectifier output DC voltage waveform that is smoothed with a capacitor. However, there is still some ripple voltage (AspenCore, n.d.).

The independent variable is the input root mean square (RMS) AC voltage. The RMS value of AC represents the same average power as produced by an equivalent DC. It thus relates the independent AC voltage variable with the dependent DC variable. It is measured on the RMS AC voltage setting of the multimeter. To manipulate RMS AC voltage, a transformer is used. A transformer is a passive electrical device that transfers electrical energy through electromagnetic induction between its primary and secondary windings. Turning its knob changes the turn ratio between the windings, which increases or decreases (steps up or steps down) AC voltage (Electrical4U, 2021).

The controlled variables are (a) diode material, (b) equivalent smoothing capacitance, (c) transformer turn ratio throughout each manipulation, (d) RMS AC voltage throughout each manipulation, (e) AC frequency, (f) resistance of the multimeter as a voltmeter, (g) resistance of the multimeter as an ammeter, and (h) the capacitors' charges before each trial. The same diodes, capacitors, transformer, and multimeter are used, and their placements are maintained. Each capacitor is discharged before each trial by short-circuiting its anode and cathode with a resistor.

4. Hypothesis.

I hypothesize that as RMS AC voltage increases, output DC will increase at an increasing rate. I think this because increasing voltage applied on the diodes will decrease the depletion region, which weakens the electric field barrier in increasing rates over time. This implies that current through the diode and thus the rectifier output DC will also increase at an increasing rate, eventually approaching infinity.

5. Materials and Procedure.

The required materials are (1) one transformer, (2) four 1N4007 rectifier diodes, (3) three 100 μF capacitors, (4) one 10 Ω resistor, (5) one multimeter, and (6) various wires as appropriate (breadboard jumper wires, alligator clips, etc.).

Before working, make sure hands are dry, to avoid the major safety issue of electrocution. First, connect a transformer to a wall socket. Make two series networks of two rectifier diodes, and connect both networks in parallel to each other on a breadboard. Connect both output wires of the transformer between both series networks of diodes, and connect a parallel network of three 100 μF capacitors to the two junctions of the diode networks (Figure 3).

Set the multimeter to the RMS AC voltmeter setting, and connect each test probe to each output wire of the transformer. Turn on the transformer, rotate its knob to the desired manipulation, and record the RMS AC voltage reading. Then, turn off the transformer and connect its output wires to the rectifier circuit. Next, set the multimeter to the DC ammeter setting (in mA), and connect the negative test probe in series to a 10 Ω load resistor. Connect the positive test probe and the other end of the load resistor to the anode and cathode of one of the smoothing capacitors, respectively (Figure 3). Turn on the transformer and record the DC reading. After, turn off the transformer, and set the multimeter to the DC voltmeter setting. For each capacitor, connect the multimeter in parallel to it, short its cathode and anode with a resistor (with sufficient resistance for it to not burn) until the multimeter displays zero volts, and make sure that the cathode, anode, and resistor wires are not touched (to avoid getting electrocuted). For five different transformer knob manipulations, repeat these steps for five trials.

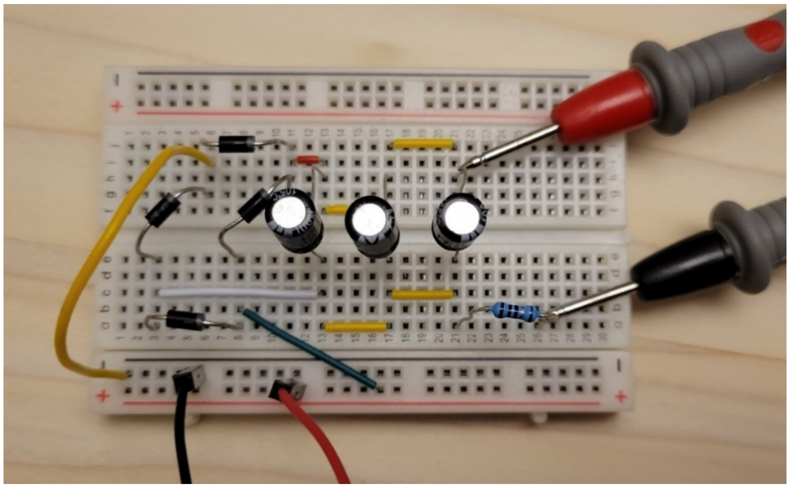


Figure 3: The circuit setup. Four rectifier diodes are on the left, in parallel to the three capacitors on the right. The bottom wires connect to the transformer, while the multimeter probes connect on the right to measure output DC.

I set the lowest RMS AC voltage to 0.00 V because that was the lowest possible voltage output of my transformer. The alternating nature of AC and the nonzero capacitance of the human body makes contacting AC more dangerous than equivalent DC. Thus, I set the highest RMS AC voltage to be low, at 4.00 V. Additionally, 4.00 V was the highest voltage rating of the solderless breadboard I am using, and also is lower than the maximum voltage limits of the diodes and capacitors used.

6. Raw Data.

The raw data I collected for my lab can be found in the table below:

RMS AC Voltage (V) ± 0.06	DC (mA)					± Uncertainty in DC (mA)
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	
0.00	0.00	0.00	0.00	0.00	0.00	0.01
1.00	0.8	0.7	0.8	0.8	0.8	0.1
2.68	11	8	9	8	10.	1
3.37	21	23	18	18	19	2
4.00	35	39	36	37	39	2

The uncertainty due to the multimeter for measuring RMS AC voltage is 0.01 V, because that was the smallest unit of measurement. There is also a 0.03 V uncertainty as the maximum observed offset of the multimeter RMS AC voltage reading from before and after 5 trials were performed for one manipulation. Furthermore, there is random error from the wall socket and transformer causing AC voltage fluctuations, which were not measured between trials. This adds 0.02 V of uncertainty. The multimeter's internal resistance, diode capacitance, and random error from temperature and humidity are negligible. Thus, the total uncertainty for my manipulated variable is 0.06 V.

The uncertainty due to the multimeter for measuring DC is 0.01 mA, as that was the smallest unit of measurement. In addition, the multimeter's DC reading constantly fluctuated, due to the systematic error of ripple voltage. As RMS AC voltage increased, this error also increased. I recorded the multimeter reading that seemed like the average of the upper and lower bounds of the fluctuations. By waiting at least 10 seconds before recording, I made sure the multimeter reading completely cycled between the upper and lower bounds. This uncertainty due to said method of measurement is equal to half of the absolute difference between the upper and lower bounds. This ranged from 0 mA to 2 mA depending on the input AC voltage. Random errors as the wire resistance and other atmospheric factors are negligible. Thus, the total uncertainty for DC rounded to one significant figure ranged from 0.01 mA to 2 mA.

7. Processed Data.

For each manipulation of RMS AC voltage, I calculated the average DC of 5 trials, as well as the corresponding uncertainties. The equations used are shown below.

$$I_{avg} = \frac{I_1 + I_2 + \cdots + I_n}{n}$$

$$\Delta I_{avg} = \frac{I_{max} - I_{min}}{n}$$

As an example, the trial data for the largest RMS AC voltage at 4.00 V was used:

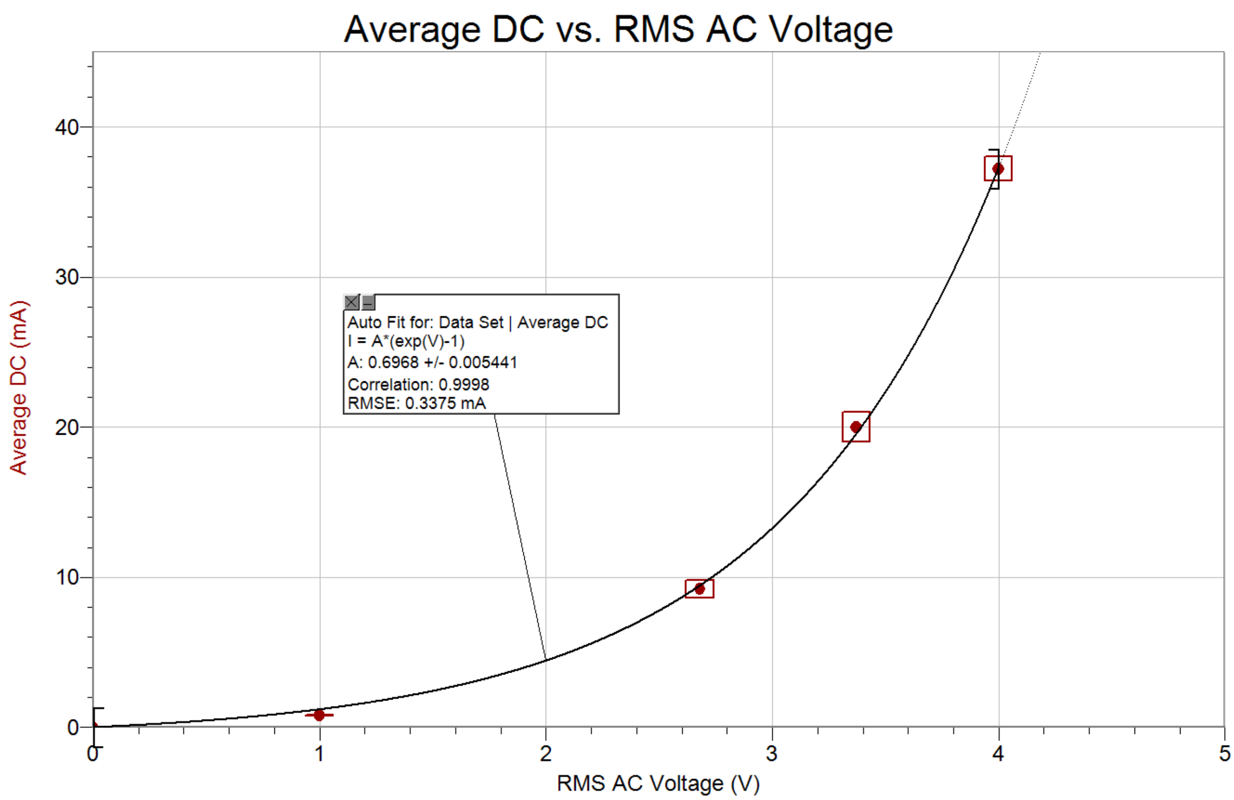
$$I_{avg} = \frac{35 + 39 + 36 + 37 + 39}{5} = \boxed{37.2\ mA}$$

$$\Delta I_{avg} = \frac{39 - 35}{5} = \boxed{0.8\ mA}$$

Using these calculations, I processed my raw data to find the average DC as well as the uncertainty of those averages, which is shown in the table below:

RMS AC Voltage (V)	Average DC (mA)	± Uncertainty in RMS AC Voltage (V)	± Uncertainty in Average DC (mA)
0.00	0.00	0.06	0.00
1.00	0.78	0.06	0.02
2.68	9.2	0.06	0.6
3.37	20.	0.06	1
4.00	37.2	0.06	0.8

I then graphed this processed data with error boxes and a curve fit, as seen below. It is noted that there exists a barely visible point on the bottom-left of the graph.



Judging by the curved-up shape of the graph, the function that fits the data could be a power or exponential fit. As voltage increases, I expect current to increase at an increasing rate, as this would conform to the exponential I-V of the non-Ohmic diodes. However, I expect current to be zero when the voltage is zero. Both a power function and a vertically translated exponential function can achieve this characteristic. I believe that a vertically translated exponential function that intersects the origin is the best fit, because it also has a negative horizontal asymptote as opposed to no asymptote for a power function. This matches my expectation that as voltage becomes more negative, current flattens out at a negative value, because unideal diodes have a nonzero reverse leakage current. Notably, the reverse breakdown of diodes for high magnitude negative voltages is ignored, but even if it was considered (as an extension to this investigation), the current between zero and the reverse breakdown voltage will still flatten out, conforming to an exponential function.

The best fit function for RMS AC voltages above the reverse breakdown voltage is thus an exponential function. I chose a base e due to its simplicity in often appearing in nature (Occam's Razor). To ensure the function intersects the origin, I applied a vertical translation of -1. Thus, with I as the DC and V as the RMS AC voltage, the best fit function is $I = A(e^V - 1)$ for some A. Using software, I found that $A = 0.6968$.

8. Linearization

From the best fit function, $I \propto e^V - 1$. To linearize the data, one is added to both sides, and then the natural log is taken on both sides, to obtain the linearized y-variable:

$$y_{lin} = \ln(I + 1)$$

The uncertainty is calculated by finding the uncertainty range of the natural logarithm using minimum and maximum raw values of I, and halving the result:

$$\Delta y_{lin} = \Delta \ln(I + 1) = \frac{\ln(I + 1 + \Delta I) - \ln(I + 1 - \Delta I)}{2}$$

$$\Delta y_{lin} = \frac{1}{2} \ln \frac{I + 1 + \Delta I}{I + 1 - \Delta I}$$

An example is shown below which uses the highest average DC:

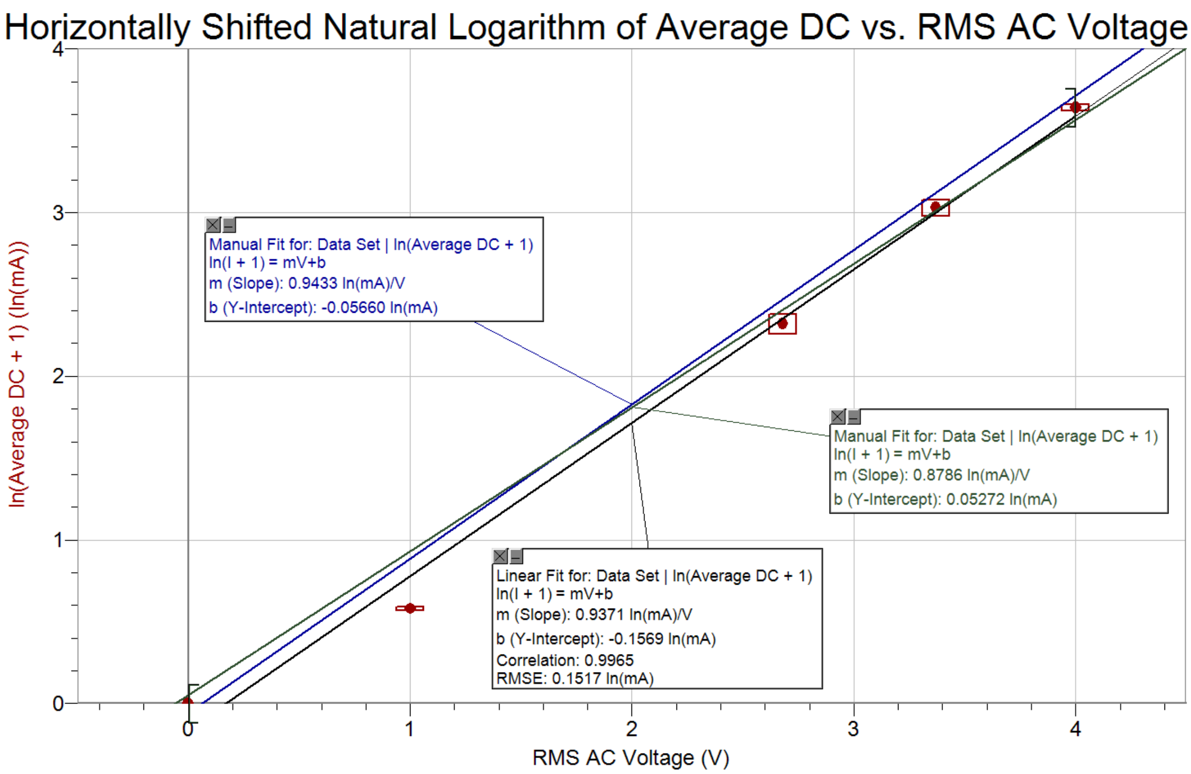
$$y_{lin} = \ln(37.2 + 1) = 3.6428355 \approx \boxed{3.64 \ln(mA)}$$

$$\Delta y_{lin} = \frac{1}{2} \ln \frac{37.2 + 1 + 0.6}{37.2 + 1 - 0.6} = 0.015708098 \approx \boxed{0.02 \ln(mA)}$$

Below is my calculated linearized data table:

RMS AC Voltage (V)	ln(Average DC + 1) (ln(mA))	± Uncertainty in RMS AC Voltage (V)	± Uncertainty in ln(Average DC + 1) (ln(mA))
0.00	0.00	0.06	0.00
1.00	0.58	0.06	0.01
2.68	2.32	0.06	0.06
3.37	3.03	0.06	0.05
4.00	3.64	0.06	0.02

This is graphed below, with maximum and minimum gradients and a best fit line:



In the linearized graph, the gradients stretch to $V < 0$ because negative voltage from the AC source is possible. Although the automatic linear fit passed through three out of five error boxes, the minimum gradient line passed through four. This mostly validates the proportionality between the horizontally shifted natural logarithm of average DC and the RMS AC voltage. The best fit line justifies the exponential nature of the original data and results in the final equation:

$$\ln(I + 1) = 0.9371V - 0.1569$$

$$I = e^{0.9371V - 0.1569} - 1 \Rightarrow \boxed{I = \frac{1}{e^{0.1569}} e^{0.9371V} - 1\ mA}$$

Although the outlier at $V = 1.00\ V$ is close to the trend line, it still demonstrates the existence of error. Other than the systematic error of ripple voltage and the random error of fluctuating AC voltages, what most likely caused the outlier was the methodologically overlooked factor of the forward voltage of the diodes, which was 1.0 V. Zero current can be conducted before the applied voltage crosses this threshold, which justifies the outlier in my data as being lower than the trendline. It also suggests that a horizontally translated exponential function with an x-intercept at the forward voltage of the diodes would better fit the data. It is noted that although RMS AC voltage was 1.00 V, the peak AC voltage was above 1.00 V, so nonzero current was measured. In a future experiment, this could be resolved by performing more manipulations of voltage that is above 1.00 V.

The consideration of error demonstrates the importance of considering all sources of uncertainty. Despite the trend being clearly of the exponential type, uncertainty impacts the analysis by presenting a range of possible functions to describe the data. Ultimately, uncertainty is crucial in allowing flexibility of analysis to model natural phenomena, given the limitations of methods of manipulation and measurement.

9. Conclusion.

Through this investigation, the answer to the question of how RMS AC voltage affects DC through a full bridge rectifier was found to be $I = \frac{1}{e^{0.1569}} e^{0.9371V} - 1\ mA$. The positive slope matches the physical expectation that increasing V would decrease the electrical field strength of the diode depletion region and cause an increase in I. Also, the function's slope increases as V increases, conforming to the exponential I-V of diodes. The horizontal asymptote at $I = -1\ mA$ also conforms to the concept of the diodes' reverse leakage current. The function approximately intersects the origin, which makes sense as zero applied voltage should cause zero current to flow.

This experiment contained many methodological issues, and can be greatly improved. The systematic error from ripple voltage caused fluctuations that made measurement of DC difficult. Using capacitors of higher capacitance would have reduced this problem. There was also random error from fluctuating AC voltages between trials, which can be reduced by using an AC generator instead of connecting a transformer to the wall socket. Aside from improving this experiment, there are also worthwhile extensions. For one, the ripple voltage and the full bridge rectifier's output voltage waveform can be measured. Different diodes can also be used to examine other diode properties. Furthermore, the rectifier's power and efficiency can be measured and analyzed to better understand how electrical energy can be used more effectively.

The equation found from this investigation is similar to the exact I-V relationship of diodes: $I = I_0\left(e^{\frac{qV}{\eta KT}} - 1\right)$ (Electrical4U, 2020). For both, I is proportional to $e^{kV} - 1$ for some constant k. This again justifies the answer to the research question, and demonstrates how the characteristics of a diode directly influence the characteristics of a full bridge rectifier. Furthermore, the exponential relationship exhibited in this investigation is similar to the exponential charging and discharging behavior of capacitors. This highlights the hidden connection in physics that diodes and capacitors are fundamentally related in their compound nature.

References

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